4-2 Operations with Matrices

Main Ideas

- Add and subtract matrices.
- Multiply by a matrix scalar.

New Vocabulary

scalar

scalar multiplication

GET READY for the Lesson

Eneas, a hospital dietician, designs weekly menus for his patients and tracks nutrients for each daily diet. The table shows the Calories, protein, and fat in a patient's meals over a three-day period.

Day	Breakfast			Lunch			Dinner		
	Calories	Protein (g)	Fat (g)	Calories	Protein (g)	Fat (g)	Calories	Protein (g)	Fat (g)
1	566	18	7	785	22	19	1257	40	26
2	482	12	17	622	23	20	987	32	45
3	530	10	11	710	26	12	1380	29	38

These data can be organized in three matrices representing breakfast, lunch, and dinner. The daily totals can then be found by adding the three matrices.

Add and Subtract Matrices Matrices can be added if and only if they have the same dimensions.

KEY CONCEPT — Addition and Subtraction of Matrices

Words If A and B are two $m \times n$ matrices, then $A + B$ is an $m \times n$ matrix in which each element is the sum of the corresponding elements of A and B. Also, $A - B$ is an $m \times n$ matrix in which each element is the difference of the corresponding elements of A and B.

Symbols

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} + \begin{bmatrix} j & k & l \\ m & n & o \\ p & q & r \end{bmatrix} = \begin{bmatrix} a+j & b+k & c+l \\ d+m & e+n & f+o \\ g+p & h+q & i+r \end{bmatrix}$$

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} - \begin{bmatrix} j & k & l \\ m & n & o \\ p & q & r \end{bmatrix} = \begin{bmatrix} a-j & b-k & c-l \\ d-m & e-n & f-o \\ g-p & h-q & i-r \end{bmatrix}$$

EXAMPLE 1 Add Matrices

a. Find $A + B$ if $A = \begin{bmatrix} 4 & -6 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 7 \\ 5 & -9 \end{bmatrix}$.

$$A + B = \begin{bmatrix} 4 & -6 \\ 2 & 3 \end{bmatrix} + \begin{bmatrix} -3 & 7 \\ 5 & -9 \end{bmatrix}$$ Definition of matrix addition

$$= \begin{bmatrix} 4 + (-3) & -6 + 7 \\ 2 + 5 & 3 + (-9) \end{bmatrix} \text{ or } \begin{bmatrix} 1 & 1 \\ 7 & -6 \end{bmatrix}$$ Simplify.

(continued on the next page)

b. **Find $A + B$ if $A = \begin{bmatrix} 3 & -7 & 4 \\ 12 & 5 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 9 \\ 4 & -6 \end{bmatrix}$.**

Since the dimensions of A are 2×3 and the dimensions of B are 2×2, you cannot add these matrices.

CHECK Your Progress

1. Find $A + B$ if $A = \begin{bmatrix} -5 & 7 \\ -1 & 12 \end{bmatrix}$ and $B = \begin{bmatrix} 11 & 3 \\ -4 & -5 \end{bmatrix}$.

EXAMPLE 2 Subtract Matrices

Find $A - B$ if $A = \begin{bmatrix} 9 & 2 \\ -4 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 6 \\ 8 & -2 \end{bmatrix}$.

$$A - B = \begin{bmatrix} 9 & 2 \\ -4 & 7 \end{bmatrix} - \begin{bmatrix} 3 & 6 \\ 8 & -2 \end{bmatrix} \quad \text{Substitution}$$

$$= \begin{bmatrix} 9 - 3 & 2 - 6 \\ -4 - 8 & 7 - (-2) \end{bmatrix} \quad \text{Subtract corresponding elements.}$$

$$= \begin{bmatrix} 6 & -4 \\ -12 & 9 \end{bmatrix} \quad \text{Simplify.}$$

CHECK Your Progress

2. Find $A - B$ if $A = \begin{bmatrix} 12 & -4 \\ -5 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & 3 \\ -3 & -2 \end{bmatrix}$.

Real-World Link

The rarest animal in the world today is a giant tortoise that lives in the Galapagos Islands. "Lonesome George" is the only remaining representative of his species (*Geochelone elephantopus abingdoni*). With virtually no hope of discovering another specimen, this species is now effectively extinct.

Source: ecoworld.com

Real-World EXAMPLE 3

ANIMALS **The table below shows the number of endangered and threatened species in the United States and in the world. How many more endangered and threatened species are there on the world list than on the U.S. list?**

Endangered and Threatened Species

Type of Animal	United States		World	
	Endangered	Threatened	Endangered	Threatened
Mammals	68	10	319	27
Birds	77	13	252	19
Reptiles	14	22	78	37
Amphibians	11	10	19	11
Fish	71	43	82	44

Source: Fish and Wildlife Service, U.S. Department of Interior

The data in the table can be organized in two matrices. Find the difference of the matrix that represents species in the world and the matrix that represents species in the U.S.

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$$\begin{array}{cccc} \textbf{World} & \textbf{U.S.} & & \textbf{Endangered Threatened} \\ \begin{bmatrix} 319 & 27 \\ 252 & 19 \\ 78 & 37 \\ 19 & 11 \\ 82 & 44 \end{bmatrix} & - \begin{bmatrix} 68 & 10 \\ 77 & 13 \\ 14 & 22 \\ 11 & 10 \\ 71 & 43 \end{bmatrix} & = & \begin{bmatrix} 319-68 & 27-10 \\ 252-77 & 19-13 \\ 78-14 & 37-22 \\ 19-11 & 11-10 \\ 82-71 & 44-43 \end{bmatrix} \end{array}$$

Subtract corresponding elements.

$$= \begin{bmatrix} 251 & 17 \\ 175 & 6 \\ 64 & 15 \\ 8 & 1 \\ 11 & 1 \end{bmatrix}$$

The first column represents the difference in the number of endangered species on the world and U.S. lists. There are 251 mammals, 175 birds, 64 reptiles, 8 amphibians, and 11 fish species in this category.

The second column represents the difference in the number of threatened species on the world and U.S. lists. There are 17 mammals, 6 birds, 15 reptiles, 1 amphibian, and 1 fish species in this category.

CHECK Your Progress

3. Refer to the data on page 169 and use matrices to show the difference of Calories, protein, and fat between lunch and breakfast.

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Scalar Multiplication You can multiply any matrix by a constant called a **scalar.** This operation is called **scalar multiplication**.

KEY CONCEPT *Scalar Multiplication*

Words The product of a scalar k and an $m \times n$ matrix is an $m \times n$ matrix in which each element equals k times the corresponding elements of the original matrix.

Symbols $k\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} ka & kb & kc \\ kd & ke & kf \end{bmatrix}$

EXAMPLE Multiply a Matrix by a Scalar

4 **If** $A = \begin{bmatrix} 2 & 8 & -3 \\ 5 & -9 & 2 \end{bmatrix}$**, find** $3A$**.**

$3A = 3\begin{bmatrix} 2 & 8 & -3 \\ 5 & -9 & 2 \end{bmatrix}$ Substitution

$= \begin{bmatrix} 3(2) & 3(8) & 3(-3) \\ 3(5) & 3(-9) & 3(2) \end{bmatrix}$ or $\begin{bmatrix} 6 & 24 & -9 \\ 15 & -27 & 6 \end{bmatrix}$ Simplify.

CHECK Your Progress

4. If $A = \begin{bmatrix} 7 & -4 & 10 \\ -2 & 6 & -9 \end{bmatrix}$, find $-4A$.

Many properties of real numbers also hold true for matrices.

CONCEPT SUMMARY ***Properties of Matrix Operations***

For any matrices A, B, and C with the same dimensions and any scalar c, the following properties are true.

Commutative Property of Addition	$A + B = B + A$
Associative Property of Addition	$(A + B) + C = A + (B + C)$
Distributive Property	$c(A + B) = cA + cB$

EXAMPLE Combination of Matrix Operations

Study Tip

Additive Identity

The matrix $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ is called a *zero matrix.* It is the *additive identity matrix* for any 2×2 matrix. How is this similar to the additive identity for real numbers?

5 **If** $A = \begin{bmatrix} 7 & 3 \\ -4 & -1 \end{bmatrix}$ **and** $B = \begin{bmatrix} 9 & 6 \\ 3 & 10 \end{bmatrix}$**, find** $5A - 2B$**.**

Perform the scalar multiplication first. Then subtract the matrices.

$$5A - 2B = 5\begin{bmatrix} 7 & 3 \\ -4 & -1 \end{bmatrix} - 2\begin{bmatrix} 9 & 6 \\ 3 & 10 \end{bmatrix}$$ Substitution

$$= \begin{bmatrix} 5(7) & 5(3) \\ 5(-4) & 5(-1) \end{bmatrix} - \begin{bmatrix} 2(9) & 2(6) \\ 2(3) & 2(10) \end{bmatrix}$$ Multiply each element in the first matrix by 5 and multiply each element in the second matrix by 2.

$$= \begin{bmatrix} 35 & 15 \\ -20 & -5 \end{bmatrix} - \begin{bmatrix} 18 & 12 \\ 6 & 20 \end{bmatrix}$$ Simplify.

$$= \begin{bmatrix} 35 - 18 & 15 - 12 \\ -20 - 6 & -5 - 20 \end{bmatrix} \text{ or } \begin{bmatrix} 17 & 3 \\ -26 & -25 \end{bmatrix}$$ Subtract corresponding elements.

CHECK Your Progress

5. If $A = \begin{bmatrix} 4 & -2 \\ 5 & -9 \end{bmatrix}$ and $B = \begin{bmatrix} 8 & 2 \\ -1 & -3 \end{bmatrix}$, find $6A - 3B$.

GRAPHING CALCULATOR LAB

Study Tip

Matrix Operations

The order of operations for matrices is similar to that of real numbers. Perform scalar multiplication before matrix addition and subtraction.

Matrix Operations

On the TI-83/84 Plus, 2nd [MATRX] accesses the matrix menu. Choose EDIT to define a matrix. Press 1 or ENTER and enter the dimensions of the matrix *A* using the ▶ key. Then enter each element by pressing ENTER after each entry. To display and use the matrix, exit the editing mode and choose the matrix under NAMES from the [MATRIX] menu.

THINK AND DISCUSS

1. Enter $A = \begin{bmatrix} 3 & -2 \\ 5 & 4 \end{bmatrix}$. What do the two numbers separated by a comma in the bottom left corner of the screen represent?

2. Enter $B = \begin{bmatrix} 1 & 9 & -3 \\ 8 & 6 & -5 \end{bmatrix}$. Find $A + B$. What is the result and why?

CHECK Your Understanding

Perform the indicated matrix operations. If the matrix does not exist, write *impossible*.

Example 1 (pp. 169–170)

1. $\begin{bmatrix} 5 & 8 & -4 \end{bmatrix} + \begin{bmatrix} 12 & 5 \end{bmatrix}$

2. $\begin{bmatrix} 12 & 6 \\ -8 & -3 \end{bmatrix} + \begin{bmatrix} 14 & -9 \\ 11 & -6 \end{bmatrix}$

Example 2 (p. 170)

3. $\begin{bmatrix} 3 & 7 \\ -2 & 1 \end{bmatrix} - \begin{bmatrix} 2 & -3 \\ 5 & -4 \end{bmatrix}$

4. $\begin{bmatrix} 4 & 12 \\ -3 & -7 \end{bmatrix} - \begin{bmatrix} 5 & 3 \\ -4 & -4 \end{bmatrix}$

Example 3 (pp. 170–171)

SPORTS For Exercises 5–7, use the table below that shows high school participation in various sports.

Sport	Males		Females	
	Schools	Participants	Schools	Participants
Basketball	17,389	544,811	17,061	457,986
Track and Field	15,221	504,801	15,089	418,322
Baseball/Softball	14,984	457,146	14,181	362,468
Soccer	10,219	349,785	9,490	309,032
Swimming and Diving	5,758	96,562	6,176	144,565

Source: National Federation of State High School Associations

5. Write two matrices that represent these data for males and females.

6. Find the total number of students that participate in each individual sport expressed as a matrix.

7. Could you add the two matrices to find the total number of schools that offer a particular sport? Why or why not?

Example 4 (p. 171)

Perform the indicated matrix operations. If the matrix does not exist, write *impossible*.

8. $3\begin{bmatrix} 6 & -1 & 5 & 2 \\ 7 & 3 & -2 & 8 \end{bmatrix}$

9. $-5\begin{bmatrix} 2 & -4 \\ -6 & 3 \\ -9 & -1 \end{bmatrix}$

Example 5 (p. 172)

Use matrices A, B, C, and D to find the following.

$A = \begin{bmatrix} 2 & 3 \\ 5 & 6 \end{bmatrix}$ $B = \begin{bmatrix} -1 & 7 \\ 0 & -4 \end{bmatrix}$ $C = \begin{bmatrix} 9 & -4 \\ -6 & 5 \end{bmatrix}$ $D = \begin{bmatrix} 2 & -5 \end{bmatrix}$

10. $A + B + C$

11. $3B - 2C$

12. $4A + 2B - C$

13. $B + 2C + D$

Exercises

Perform the indicated matrix operations. If the matrix does not exist, write *impossible*.

14. $\begin{bmatrix} 4 \\ 1 \\ -3 \end{bmatrix} + \begin{bmatrix} 6 \\ -5 \\ 8 \end{bmatrix}$

15. $\begin{bmatrix} -11 & 4 \\ -3 & 6 \end{bmatrix} + \begin{bmatrix} -2 & -5 \\ 5 & -3 \end{bmatrix}$

HOMEWORK HELP	
For Exercises	See Examples
14–17	1
18–21	2
22–24	3
25, 26	4
27, 28	5

Perform the indicated matrix operations. If the matrix does not exist, write impossible.

16. $\begin{bmatrix} -5 & 2 & -1 \end{bmatrix} + \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$

17. $\begin{bmatrix} 2 & 5 & 3 \\ -7 & -1 & 11 \\ 4 & -4 & 0 \end{bmatrix} + \begin{bmatrix} -9 & 2 & -5 \\ 1 & 6 & -3 \\ -9 & -12 & 8 \end{bmatrix}$

18. $\begin{bmatrix} -5 & 7 \\ 6 & 8 \end{bmatrix} - \begin{bmatrix} 4 & 0 & -2 \\ 9 & 0 & 1 \end{bmatrix}$

19. $\begin{bmatrix} 12 & 0 & 8 \\ 9 & 15 & -11 \end{bmatrix} - \begin{bmatrix} -3 & 0 & 4 \\ 9 & 2 & -6 \end{bmatrix}$

20. $\begin{bmatrix} 3 \\ -8 \\ -2 \end{bmatrix} - \begin{bmatrix} -4 \\ 5 \\ -2 \end{bmatrix}$

21. $\begin{bmatrix} -9 & 2 & -7 \\ 8 & 10 & 3 \\ -7 & 4 & 15 \end{bmatrix} - \begin{bmatrix} -1 & 3 & 6 \\ -7 & -3 & 5 \\ 2 & 11 & -4 \end{bmatrix}$

BUSINESS **For Exercises 22–24, use the following information.** An electronics store records each type of entertainment device sold at three of their device sold at three of their branch stores so that they can monitor their purchases of supplies. Two weeks of sales are shown in the spreadsheets at the right.

	A	B	C	D	E
1	Week 1	Televisions	DVD players	Video game units	CD players
2	Store 1	325	215	147	276
3	Store 2	294	221	79	152
4	Store 3	175	191	100	146

	A	B	C	D	E
1	Week 2	Televisions	DVD players	Video game units	CD players
2	Store 1	306	162	145	257
3	Store 2	258	210	84	165
4	Store 3	188	176	99	112

22. Write a matrix for each week's sales.

23. Find the sum of the two weeks' sales expressed as a matrix.

24. Express the difference in sales from Week 1 to Week 2 as a matrix.

Perform the indicated matrix operation. If the matrix does not exist, write *impossible*.

25. $-2\begin{bmatrix} 2 & -4 & 1 \\ -3 & 5 & 8 \\ 7 & 6 & -2 \end{bmatrix}$

26. $3\begin{bmatrix} 5 & -3 \\ -10 & 8 \\ -1 & 7 \end{bmatrix}$

27. $5\begin{bmatrix} 0 & -1 & 7 & 2 \end{bmatrix} + 3\begin{bmatrix} 5 & -8 & 10 & -4 \end{bmatrix}$

28. $5\begin{bmatrix} 1 \\ -1 \\ -3 \end{bmatrix} + 6\begin{bmatrix} -4 \\ 3 \\ 5 \end{bmatrix} - 2\begin{bmatrix} -3 \\ 8 \\ -4 \end{bmatrix}$

Use matrices *A*, *B*, *C*, and *D* to find the following.

$A = \begin{bmatrix} 5 & 7 \\ -1 & 6 \\ 3 & -9 \end{bmatrix}$ $B = \begin{bmatrix} 8 & 3 \\ 5 & 1 \\ 4 & 4 \end{bmatrix}$ $C = \begin{bmatrix} 0 & 4 \\ -2 & 5 \\ 7 & -1 \end{bmatrix}$ $D = \begin{bmatrix} 6 & 2 \\ 9 & 0 \\ -3 & 0 \end{bmatrix}$

29. $A + B$ **30.** $D - B$ **31.** $4C$

32. $6B - 2A$ **33.** $3C - 4A + B$ **34.** $C + \frac{1}{3}D$

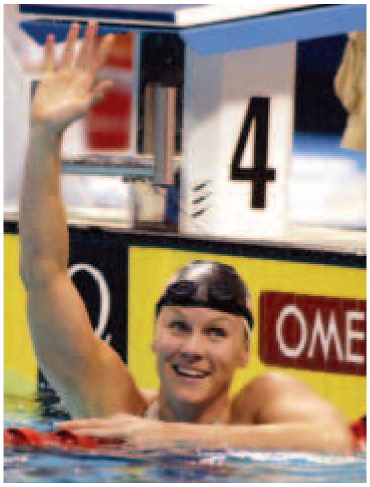
Perform the indicated matrix operation. If the matrix does not exist, write *impossible.*

35. $\begin{bmatrix} 1.35 & 5.80 \\ 1.24 & 14.32 \\ 6.10 & 35.26 \end{bmatrix} + \begin{bmatrix} 0.45 & 3.28 \\ 1.94 & 16.72 \\ 4.31 & 21.30 \end{bmatrix}$

36. $8\begin{bmatrix} 0.25 & 0.5 \\ 0.75 & 1.5 \end{bmatrix} - 2\begin{bmatrix} 0.25 & 0.5 \\ 0.75 & 1.5 \end{bmatrix}$

37. $\frac{1}{2}\begin{bmatrix} 4 & 6 \\ 3 & 0 \end{bmatrix} - \frac{2}{3}\begin{bmatrix} 9 & 27 \\ 0 & 3 \end{bmatrix}$

38. $5\begin{bmatrix} \frac{1}{2} & 0 & 1 \\ 2 & \frac{1}{3} & -1 \end{bmatrix} + 4\begin{bmatrix} -2 & \frac{3}{4} & 1 \\ \frac{1}{6} & 0 & \frac{5}{8} \end{bmatrix}$



Real-World Link

Jenny Thompson won her record setting twelfth Olympic medal by winning the silver in the 4 × 100 Medley Relay at the 2004 Athens Olympics.

Source: athens2004.com

SWIMMING For Exercises 39–41, use the table that shows some of the world, Olympic, and U.S. women's freestyle swimming records.

Distance (meters)	World	Olympic	U.S.
50	24.13 s	24.13 s	24.63 s
100	53.52 s	53.52 s	53.99 s
200	1:56.54 min	1:57.65 min	1:57.41 min
800	8:16.22 min	8:19.67 min	8:16.22 min

Source: hickoksports.com

39. Find the difference between U.S. and World records expressed as a column matrix.

40. Write a matrix that compares the total time of all four events for World, Olympic, and U.S. record holders.

41. In which events were the fastest times set at the Olympics?

RECREATION For Exercises 42 and 43, use the following price list for one-day admissions to the community pool.

42. Write the matrix that represents the additional cost for nonresidents.

43. Write a matrix that represents the difference in cost if a child or adult goes to the pool after 6:00 P.M.

Daily Admission Fees

Residents

Time of day	Child	Adult
Before 6:00 P.M.	$3.00	$4.50
After 6:00 P.M.	$2.00	$3.50

Nonresidents

Time of day	Child	Adult
Before 6:00 P.M.	$4.50	$6.75
After 6:00 P.M.	$3.00	$5.25

EXTRA PRACTICE

See pages 897, 929.

Math Online

Self-Check Quiz at algebra2.com

H.O.T. Problems

44. CHALLENGE Determine values for each variable if $d = 1$, $e = 4d$, $z + d = e$, $f = \frac{x}{5}$, $ay = 1.5$, $x = \frac{d}{2}$, and $y = x + \frac{x}{2}$.

$$a\begin{bmatrix} x & y & z \\ d & e & f \end{bmatrix} = \begin{bmatrix} ax & ay & az \\ ad & ae & af \end{bmatrix}$$

45. OPEN ENDED Give an example of two matrices whose sum is a zero matrix.

46. CHALLENGE For matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, the *transpose* of A is $A^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$.

Write a matrix B that is equal to its transpose B^T.

47. *Writing in Math* Use the data on nutrition on page 169 to explain how matrices can be used to calculate daily dietary needs. Include three matrices that represent breakfast, lunch, and dinner over the three-day period, and a matrix that represents the total Calories, protein, and fat consumed each day.

STANDARDIZED TEST PRACTICE

48. ACT/SAT Solve for x and y in the matrix equation $\begin{bmatrix} x \\ 7 \end{bmatrix} + \begin{bmatrix} 3y \\ -x \end{bmatrix} = \begin{bmatrix} 16 \\ 12 \end{bmatrix}$.

A $x = -5, y = 7$

B $x = 7, y = 3$

C $x = 7, y = 5$

D $x = 5, y = 7$

49. REVIEW What is the equation of the line that has a slope of 3 and passes through the point (2, –9)?

F $y = 3x + 11$

G $y = 3x - 11$

H $y = 3x + 15$

J $y = 3x - 15$

Spiral Review

State the dimensions of each matrix. (Lesson 4-1)

50. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

51. $\begin{bmatrix} 2 & 0 & 3 & 0 \end{bmatrix}$

52. $\begin{bmatrix} 5 & 1 & -6 & 2 \\ -38 & 5 & 7 & 3 \end{bmatrix}$

53. $\begin{bmatrix} 7 & -3 & 5 \\ 0 & 2 & -9 \\ 6 & 5 & 1 \end{bmatrix}$

54. $\begin{bmatrix} 8 & 6 \\ 5 & 2 \\ -4 & -1 \end{bmatrix}$

55. $\begin{bmatrix} 7 & 5 & 0 \\ -8 & 3 & 8 \\ 9 & -1 & 15 \\ 4 & 2 & 11 \end{bmatrix}$

Solve each system of equations. (Lesson 3-5)

56. $2a + b = 2$
$5a = 15$
$a + b + c = -1$

57. $r + s + t = 15$
$r + t = 12$
$s + t = 10$

58. $6x - 2y - 3z = -10$
$-6x + y + 9z = 3$
$8x - 3y = -16$

Solve each system by using substitution or elimination. (Lesson 3-2)

59. $2s + 7t = 39$
$5s - t = 5$

60. $3p + 6q = -3$
$2p - 3q = -9$

61. $a + 5b = 1$
$7a - 2b = 44$

SCRAPBOOKS **For Exercises 62 and 63, use the following information.** (Lesson 2-7)
Ian has $6.00, and he wants to buy paper for his scrapbook. A sheet of printed paper costs 30¢, and a sheet of solid color paper costs 15¢.

62. Write and graph an inequality that describes this situation.

63. Does Ian have enough money to buy 14 pieces of each type of paper? Explain.

GET READY for the Next Lesson

Name the property illustrated by each equation. (Lesson 1-2)

64. $\frac{7}{9} \cdot \frac{9}{7} = 1$

65. $7 + (w + 5) = (7 + w) + 5$

66. $3(x + 12) = 3x + 3(12)$

67. $6(9a) = 9a(6)$